Analytical Methods for Predicting Subsidence Above Solution-Mined Cavities

C.-Y. Chang
Keshavan Nair
Woodward-Lundgren & Associates,
Oakland, California

ABSTRACT

A general rational approach is described for analyzing complex problems dealing with subsidence prediction. Qualitative comparison of results obtained from the linear elastic analysis with observed subsidence phenomena indicated that the computed subsidence was generally much smaller than the observed subsidence and that the subsidence extended over a larger area than the actual subsidence in the field. This conclusion led to the necessity of developing techniques for computing the subsidence that may result from failure in the surrounding rock mass in the form of (1) rock failure in the immediate vicinity of the cavity or (2) failure along certain geologic discontinuities such as faults, joints, or bedding planes.

A computer program using finite element techniques has been developed to analyze the conditions that are created in a rock mass surrounding a solution mined cavity with the object of predicting the subsidence. The program, in addition to the ability to conduct a linear elastic analysis, can account for the inability of the rock mass to withstand tensile stresses and the occurrence of slippage along bedding planes. The formulation of the analysis and the capabilities of the program are discussed in the paper. The program does not consider time-dependent problems; this is treated in a companion paper by Nair et al. of this Symposium.

The analytical methods developed were employed to analyze two case histories of solution mined cavities. A comparison with observed subsidence indicates that the method can be utilized to predict subsidence above solution mined cavities with reasonable accuracy.

INTRODUCTION

The creation of an underground opening causes movements and induces stress changes in the surrounding rock mass. The movements are manifested as subsidence at the ground surface above the opening. The subsidence and the associated horizontal strain are of concern to the mining industry because of the possible damage that can result to structures located on the ground surface. Therefore, in the planning and conducting of mining operations, it is desirable to develop a reliable method of forecasting the probable subsidence which may occur above mined areas.

There are a number of factors that are known to influence the magnitude and the nature of the subsidence occurring above mined areas. The most significant factors are (1) the rock profile; (2) the rock properties; (3) the location, size, and shape of the opening; (4) the presence of faults, shear zones, bedding planes, and other geological discontinuities; (5) the presence of other openings; (6) the initial stress state; and (7) any artificial support in the openings including packing or filling. Because of the complexity of the problem, a general approach was considered necessary for the development of a rational method of subsidence prediction. This approach, which is summarized in Figure 1, consists of the following major steps:

Establishment of objectives and performance criteria

In any design process, it is necessary to establish the objectives of the design and to translate these objectives into performance criteria (measures of performance). For example, the performance criteria in subsidence problems may be in terms of limiting the total or differential settlement and the horizontal strain at the surface. In defining the response variables, it should be recognized that these should be in terms of the performance criteria in order that a comparison can be made.

Definition of input and output (response) variables

Based on the available geologic information, the initial stress state in the rock mass is defined and the stress distribution on the cavity face which determines the boundary loads that will be applied to the system to simulate the creation of the opening is assumed. In addition to loads, other input variables are environmental and construction factors. The output variables for most problems in rock mechanics are stress, strain, and displacement.

Physical description of the system

The description of the system consists of the following: (a) the size and shape of the opening; (b) its location below the surface; (c) the profile and the distribution of geological discontinuities (e.g., fissures, faults, joints, cleavages and bedding planes) of the surrounding rock; (d) mechanical properties of the surrounding rock, and (e) support schemes used to maintain stability.

Determination of the response of the system

This requires (a) the development of a model for the system and (b) the use of analytical and experimental techniques to determine the response of the model to the prescribed inputs. Two general approaches may be employed to determine the response of the system from the defined input and the described system. They are the testing approach and the macroanalytical approach. The latter involves the development of a mathematical model for the system. It is the macroanalytical approach that forms the basis for the majority of the existing design methods in engineering practice and was used in the study reported herein.

Decision on acceptability of subsidence

Based on field experience, if the predicted results appear reasonable, the output of the system should be compared with performance criteria to see if surface subsidence and horizontal strain are within allowable limits to prevent damage to structures situated in the vicinity of the mined area.

Improvement of prediction techniques

This consists of monitoring the performance of the system and comparing it with predictions. Such comparisons are essential to the development of improved predictive techniques.

The scope of this paper involves Steps 2 through 4. In addition, a number of case history studies where field subsidence information is available were studied for the purpose of establishing the feasibility of using the techniques developed in this investigation for predicting subsidence.

Many of the variables influencing the subsidence were studies by Nair and Chang (1969a). They were the size, the depth of the cavity, the initial stress state, and the variation in modulus within bedded deposits. It has been shown that most of these variables may be accounted for when the finite element technique is utilized to compute the subsidence due to underground cavities. Qualitative comparison of analytic results with observed subsidence phenomena indicated, however, that the computed subsidence was generally much smaller than the observed subsidence, and that the subsidence extended over a larger area than the actual subsidence in the field. The major cause of the discrepancy was a result of the assumption made in the previous analytical studies that the rock surrounding an opening is, and acts as, a continuous mass. It is known that in the presence of a stress field, the existence of geologic features in a rock mass such as fissures, faults, joints, cleavages and bedding planes can cause large deformations and failures. Furthermore, the geologic features mentioned above tend to localize the deformation to the area above the opening, by limiting the continuum action of the surrounding rock mass. Consequently, the deformation pattern obtained from the analytical study treating the rock mass as a true continuum will underestimate the deflection near the opening and the influence of the opening will extend to a greater distance. Therefore, it was considered logical that techniques be developed for computing the increased subsidence that may result from failure in the surrounding rock mass in the form of (1) rock failure in the immediate vicinity of the opening due to critical stress conditions, or (2) failure along certain geological discontinuities such as faults, joints or bedding planes, or due to the presence of fissures in the rock mass. It is the development and utilization of analytical techniques to include these factors that is the primary concern of this paper.

FAILURE CRITERIA

As discussed previously, this study is concerned with the subsidence above mined areas due to failure in the surrounding rock mass. It is, therefore, appropriate to review briefly the types of rock failure that are pertinent to the prediction of subsidence from solution mining operations. Four possible types of rock failure are discussed in the following paragraphs.

Compressive yielding

A rock mass when subjected to a certain stress state in the field may reach a point where yielding* may occur without change of external loads. For purely cohesive isotropic materials, the yield criterion is defined by the following stress state:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = k^2 \qquad (1)$$

in which σ_1, σ_2, σ_3 are the principal stresses and k a limiting value deduced from uniaxial tests.

A plasticity criterion which includes friction effects,

*Yielding is considered under failure for the sake of simplicity. It is recognized that yielding does not imply failure in the usual sense.

proposed by Drucker (1952) is considered to be more representative of the behavior of rocks. This is expressed by the following equation:

$$\alpha(\sigma_1 + \sigma_2 + \sigma_3) + \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = k \quad (2)$$

in which material properties α and k may be determined empirically. Other ideal plasticity criteria can be used. The yielding of rock will primarily occur in the immediate vicinity of the opening where stress concentrations are likely to exist.

Brittle fracture-no tension

It is assumed that rock is incapable of withstanding significant tensile stresses. This is due primarily to the presence of cracks and fissures which exist in rock in its natural state. Zienkiewicz, et al. (1968) have developed a method of stress analysis, which uses finite element techniques, to include this characteristic of rock. The method has been used with success to analyze stress distributions for a number of rock mechanics case studies.

Slippage along bedding planes

On the basis of observations in shafts on rock movements due to mining, Mohr (1951, 1958) has shown that the shearing stresses may overcome the frictional resistance between two beds or within the same bed. The relative horizontal displacement of the rock beds resulting from this phenomenon has been found to vary from a few centimeters up to 50 centimeters. This type of relative displacements has been observed over a larger lateral extent and in the same rock bed. Figure 2a shows rock movements above a mined area as observed from shafts. Figure 2b illustrates large relative horizontal displacements occurring within strata as observed from two neighboring shafts. This type of rock movement is considered likely to occur in laminated sedimentary rocks, e.g., shale, which have a small resistance to movements along bedding planes.

Slippage along other geological discontinuities

When the opening is situated in the area where joints or faults are prevalent, the surface subsidence will be substantially affected by the existence of these geological discontinuities. Examples of this type of subsidence have been reported by Deere (1961) and Lee and Strauss (1969). The analysis for this type of failure can only be performed when the actual distribution of these geological discontinuities are ascertained. The shear strength along the joint surface has been studied in detail by Patton (1966) and Deere et al. (1966). They have shown that the shear strength of a rock surface is a function of cohesion c, angle of internal friction ϕ, and the roughness of the joint surface. The roughness as shown in Figure 3 may arise from undulations in the joint surface itself or from

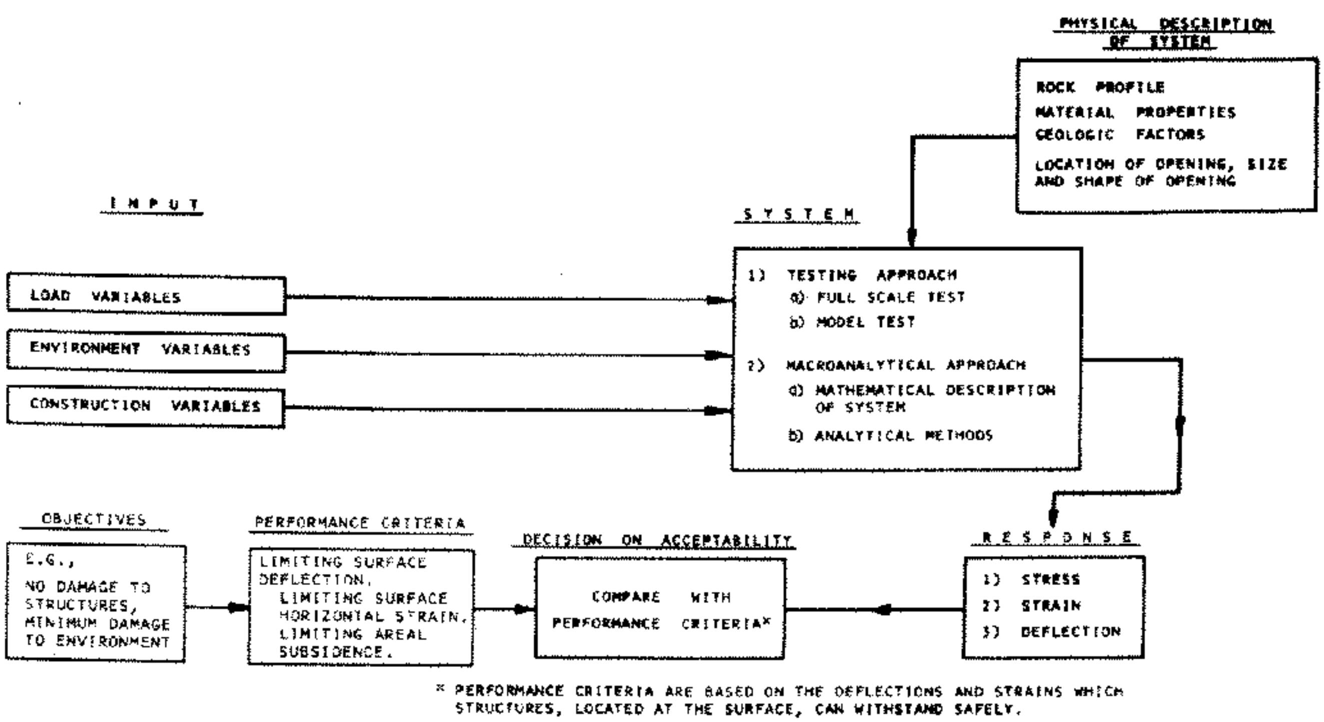

Figure 1. A general approach to the subsidence problem.

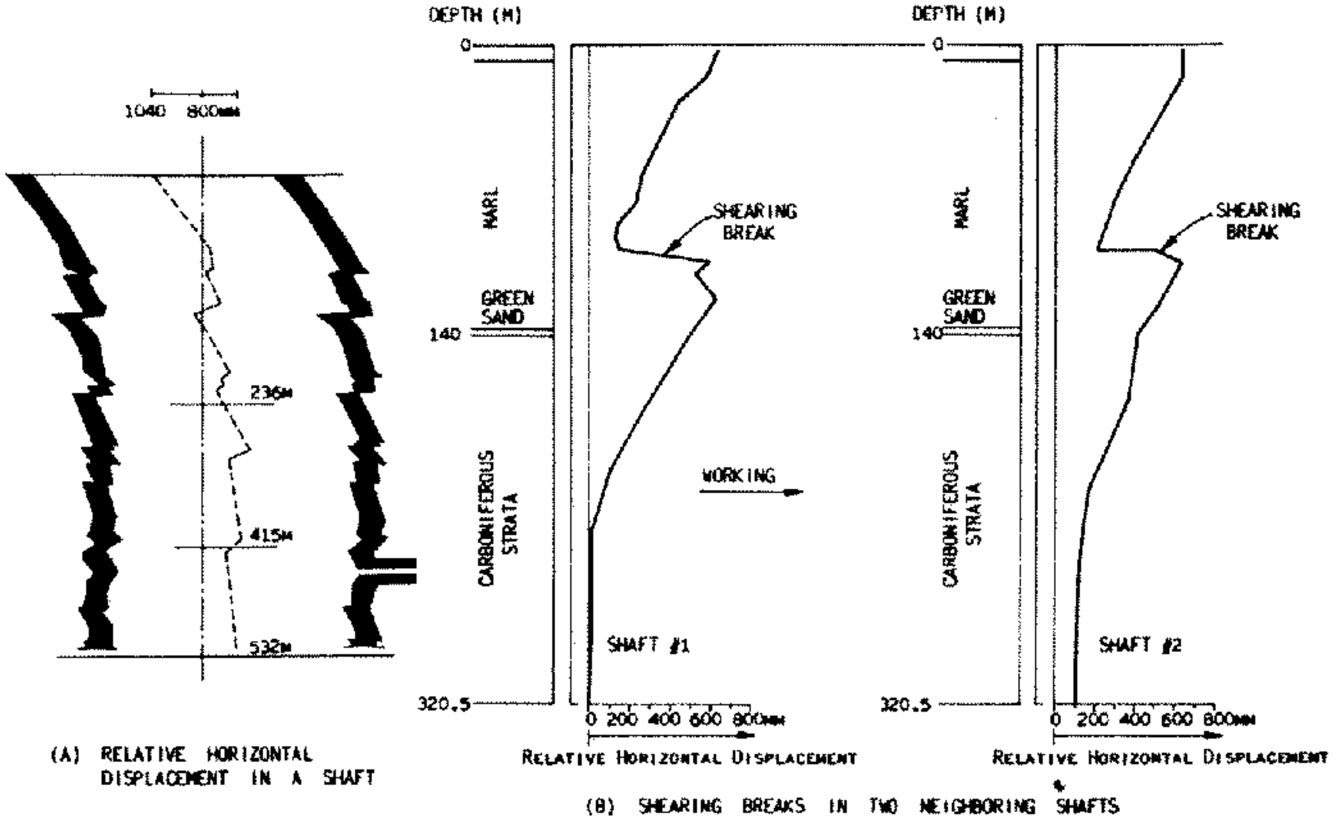

Figure 2. Field observations of horizontal movements of strata from shafts affected by mining operation (after Mohr, 1951, 1958).

steps in the potential failure plane surface resulting from the intersection of two or more joint sets. The amount of roughness may be measured by an average angle ϕ_r between the undulations on the joint surface and the direction of sliding along the joint (see Figure 3). The influence of joint roughness on the shear strength of a rock surface can be taken into account by increasing the friction angle of the joint surface by ϕ_r. Consequently, the effective friction angle ϕ_a of a rough joint surface is given by

$$\phi_a = \phi + \phi_r \quad (3)$$

The shear strength of a rough joint may be expressed by

$$\tau = c + \sigma \tan(\phi + \phi_r) \quad (4)$$

in which τ is the shear strength, and σ the normal stress acting on the sliding plane. It should be noted that the shear strength expressed by Eq. (4) may also be used as a criterion for shear breaking along bedding planes.

On the basis of experience from other studies (Reyes and Deere, 1966, and Nair, 1969b), it can be concluded that the increased deformation that accompanies yielding exerts a minor influence on the subsidence pattern. It was therefore concluded that the inclusion of the effect of yielding would not lead to subsidence predictions that differed significantly from the elastic analysis. Consequently, in this investigation, only the inability of rock to withstand tension and the phenomena of slippage along bedding planes were studied. It should be recognized that the methods of analysis used for investigating slippage

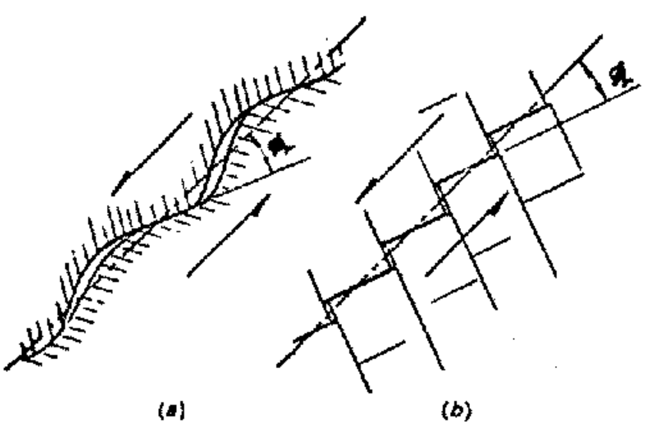

Figure 3. Types of sliding surface roughness; (a) surface roughness, (b) stepped joints.

along bedding planes are also applicable for slippage along other discontinuities.

DEVELOPMENT AND PRELIMINARY EVALUATION OF METHODS FOR PREDICTING SUBSIDENCE

In this section, analytical methods for predicting subsidence which include the capability of accounting for the inability of rock to withstand tensile stresses and the possibility of slippage along bedding planes are described. Examples were analyzed to determine the significance of these phenomena on the prediction of subsidence.

"No tension" analysis

When numerous cracks and fissures are present in a rock mass, it has been assumed that the rock is incapable of withstanding tensile stresses. A procedure for modelling this nonlinear behavior has been presented by Zienkiewicz, et al. (1968). This method, called "no tension" or "stress transfer" analysis, consists of five essential steps:

(a) Assign initial stresses to the rock mass, and calculate boundary loads required on the cavity face to simulate the creation of the opening.

(b) Analyze the problem as an elastic case. Add the induced changes in stress to the initial stresses and compute the principal stresses.

(c) Determine elements in which tension exists. As the material is assumed incapable of sustaining tensions, the calculated tensile principal stresses are eliminated without permitting any point in the structure to displace. In order to maintain equilibrium, external equivalent nodal point forces are calculated and temporarily applied to the structure.

(d) The elastic analysis is repeated to remove the balancing nodal point forces and the check for tensile stresses is repeated.

(e) If, at the end of stage (d), the tensile principal stresses are still in existence, steps (c) and (d) are repeated until all tensile stresses are reduced to an acceptable level.

In Step (c), when the linear elastic solution indicates that the rock is subjected to tensile stresses greater than the tensile strength, the rock is assumed to be fractured and incapable of transferring stresses between two orthogonal directions. To include this effect, a correction has to be made on the stress before the stress transfer process is performed. In the case where both principal tensile stresses σ_x' and σ_y' are to be transferred, the "corrected" stresses are given by

$$\sigma_x = (1 + \nu^2 + \ldots)\, \sigma_x' + \nu(1 + \nu^2 + \ldots)\, \sigma_y' \quad (5)$$

$$\sigma_y = (1 + \nu^2 + \ldots)\, \sigma_y' + \nu(1 + \nu^2 + \ldots)\, \sigma_x' \quad (6)$$

Slippage along bedding planes

Bedding planes are likely to be present in sedimentary rocks in which the majority of the solution and coal mining is carried out. Shear strength along bedding planes is considered to be weaker than that of intact rocks. As the shear stress overcomes the shear strength along these planes due to changes in stress induced by the creation of underground cavities, slippage may occur along these rock beds. This type of behavior may cause changes in the subsidence profile calculated by the elastic analysis of the underground cavity assuming the rock to be a continuum. The two major difficulties of incorporating this aspect of rock behavior into an analytical model for predicting field behavior are (1) the determination of the number and location of the bedding planes, and (2) the representation of the behavior of these bedding planes, i.e., assigning the stress-strain behavior and the strength of these weak planes. The number and location of the weak planes may be approximately estimated from 3-D velocity logs taken in the mined areas. It is, however, difficult to determine the stress-strain behavior of these bedding planes. As a first step to study the effects of the presence of the weak bedding planes in the rock mass above the cavity on the subsidence profile, it is assumed in this study that these bedding planes were very weak in resisting shear stress along the bedding planes and would yield upon creation of an opening.

Illustrative problem

A rock profile with the size and location of the cavity shown in Figure 4 was analyzed to study the effects of no-tension rock characteristics and the presence of bedding planes on the subsidence profile. Three weak bedding planes were assumed, and their locations are shown in Figure 4. These weak planes were assigned to a very low shear modulus and, therefore, offered very little resistance to movement along the plane. The result of the analysis indicates that the vertical subsidence at the centerline of the cavity is three times as large as the calculated without the presence of the bedding planes. Furthermore, the shape of the subsidence profile indicates a greater localization of the subsidence to an area above the cavity than for the elastic continuum case. This shape is in general agreement with that observed in the field. The subsidence profile obtained from the analysis is presented in Figure 5.

The results of the analysis indicate that the presence of weak bedding planes above the cavity has a major effect on the magnitude and distribution of the subsidence. It may be noted that a "no-tension" analysis, i.e., assuming that the rock mass in its natural state is incapable of sustaining tensile stresses, provides a slight increase in subsidence.

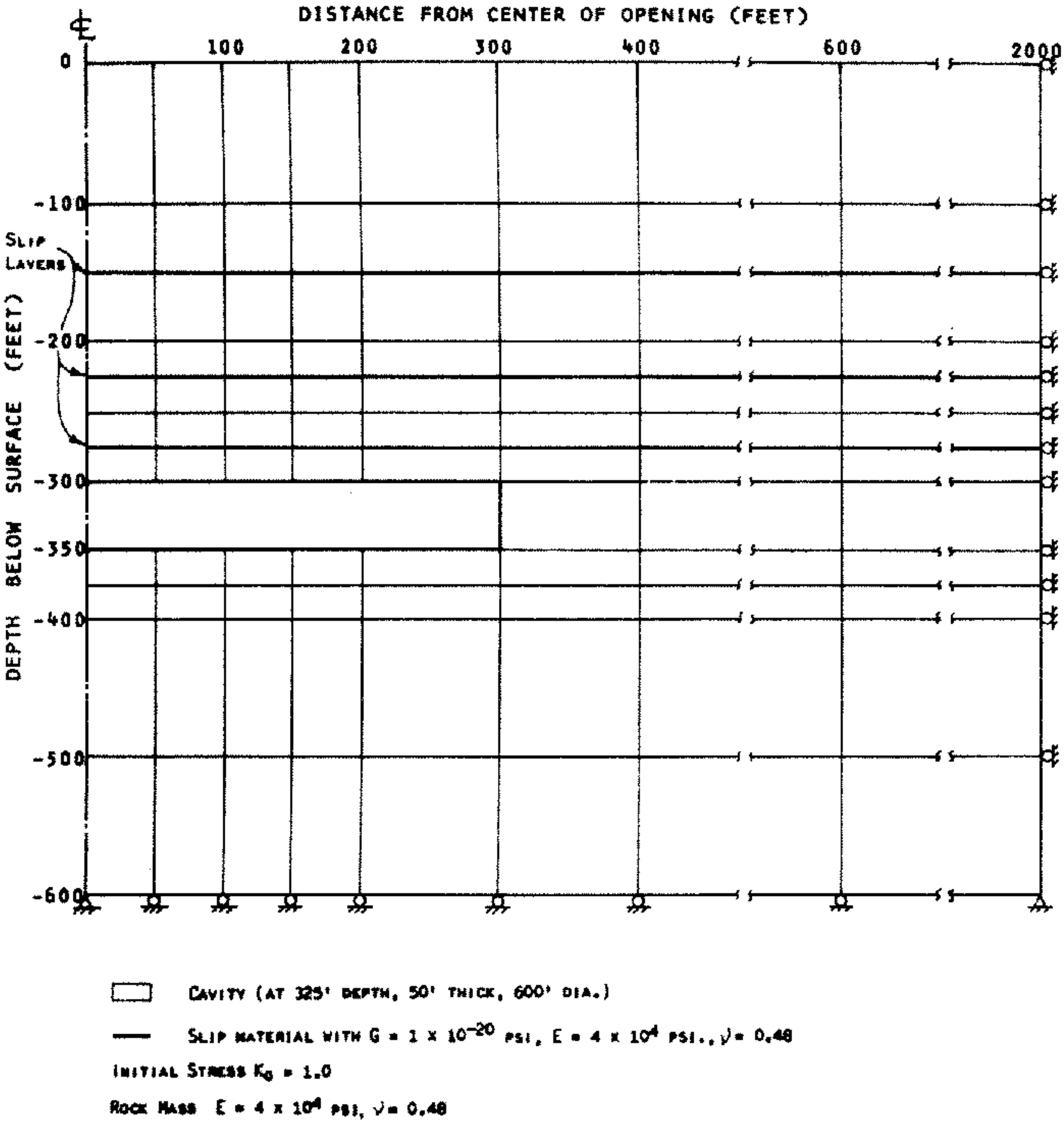

Figure 4. Finite element mesh with three layers of slip material.

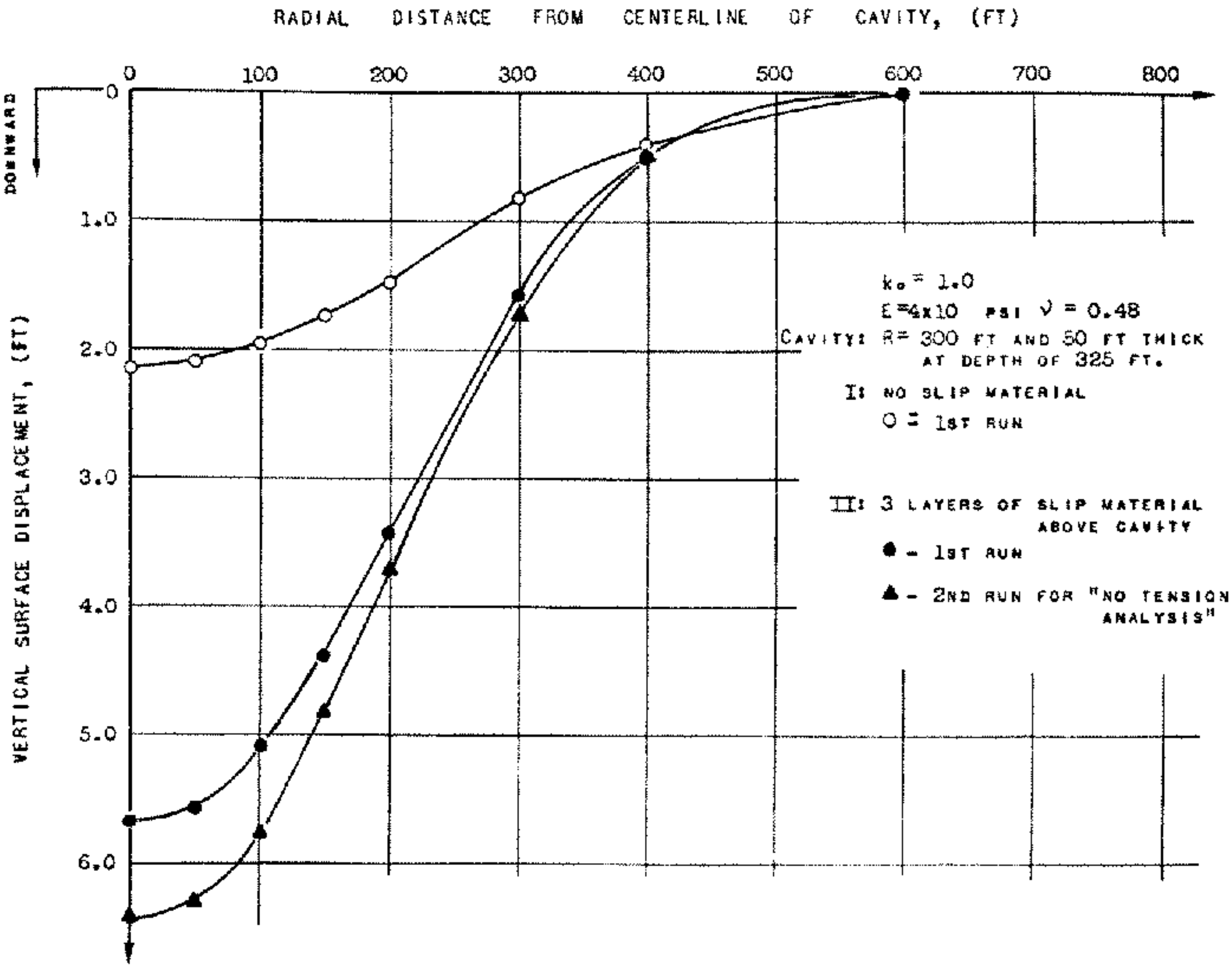

Figure 5. Vertical subsidence profiles.

Both the "no-tension" analysis and the possibility of slip along bedding planes were incorporated into a finite element program for the analysis of axisymmetric elastic solids. This program was utilized for analyzing selected case histories where subsidence records were available to enable a comparison to be made with the analytical predictions.

ANALYSIS OF CASE HISTORIES

As discussed above, methods of analyses incorporating the possibility of slippage along bedding planes offer a promising approach to the prediction of subsidence. Two case histories where field measurements were available were analyzed. The geological conditions, the approximate cavity size and location, and the observed subsidence for the two case studies are briefly described in the following paragraphs.

Case I

Geological conditions. The salt zone called Syracuse salt averages 110 feet in thickness and is at an average depth of 2270 feet below the surface. It is composed of three beds: an upper salt bed 10 feet to 20 feet in thickness, an intermediate shale zone approximately 40 feet thick, and a lower salt bed averaging 50 feet in thickness. The Syracuse salt is overlain by approximately 500 feet of upper Silurian sedimentary rocks consisting of shaley to massive anhydrite, dolomite, and limestone with some interbedded shale. The Silurian rocks are overlain in turn by 1660 feet of Devonian sedimentary rocks. The lower 140 feet comprises the Onondaga limestone. The rest consists primarily of black shales with some interbedded flaggy sandstone and occasional thin limestone beds. The idealized rock profile with assumed rock properties based on information from another location is shown in Figure 6. There was no information from which the number and the location of the weak bedding planes could be ascertained. Three weak bedding planes located at the depths of 210 feet, 1000 feet, and 1800 feet were assumed. The locations of these weak planes are shown in Figure 6.

Geometry of cavity. Because neither the vertical extent of the salt producing interval nor the horizontal extent of solution mining operations was known, the exact location and geometry of the cavity cannot be determined. From a rough estimate of total production to the date (August, 1959) when the field measurements were available (5–1/2 million tons), it was estimated that a thickness of 15 to 18 feet of salt within the area of producing wells was removed. The area of producing wells was approximately 2000 feet by 4000 feet. A number of cases were analyzed for the assumed idealized rock profile with a cavity 20 feet thick and 1000 feet, 2000 feet, and 3000 feet in radius. These cases are summarized in Table I.

Observed subsidence profile. The field measurements obtained indicate that subsidence of a magnitude of 2–1/2

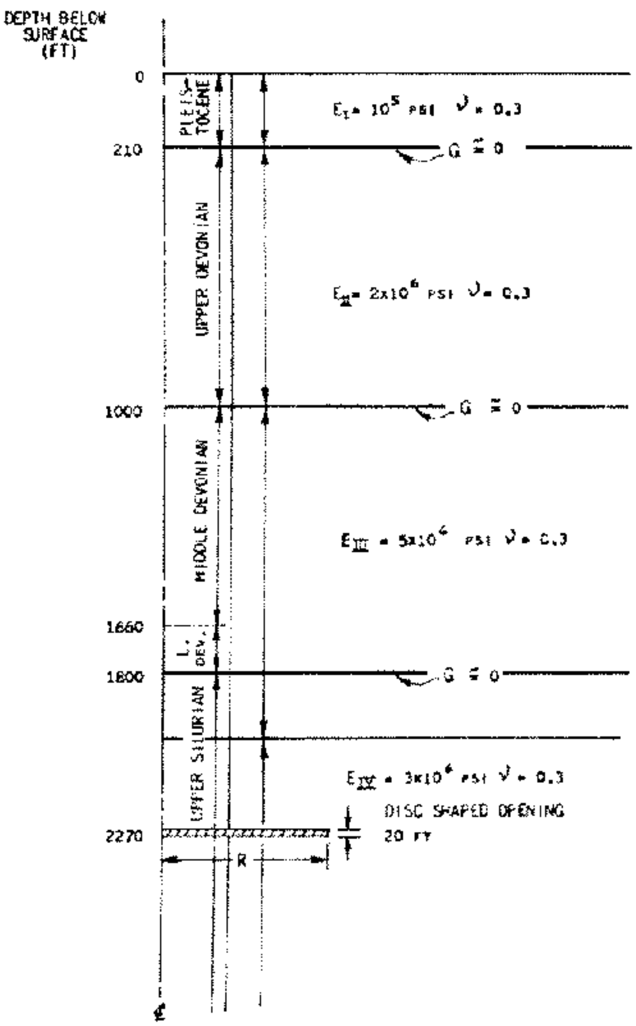

Figure 6. Idealized rock profile, Case I.

TABLE I

Analysis—Case I

Height of Cavity (ft.)	Radius of Cavity (ft.)	Slip Layers	K_o	γ pcf
20	1000	3 layers	1.0	144
20	2000	3 layers	1.0	144
20	2000	None	1.0	144
20	3000	3 layers	1.0	144

feet, and horizontal compression of the ground surface in the immediate plant area of as much as 4 inches per 100 feet, had occurred in a period between 1939 and 1958. It was observed that the subsidence was localized within the mined area. Away from the center of the mined area, the magnitude of subsidence decreased gradually.

In addition to subsidence observed in the field, caving in some of the wells was noted. The height of caving above the salt roof ranged from 40 to 122 feet indicating that the shale bed separating the two salt beds has caved in.

Comparison of computed and observed subsidence profiles. The results from the analytical studies conducted for Case I are presented in Figure 7 and Table II. An average subsidence profile measured in the field is also plotted in Figure 7 for comparison. For the cases in which three weak planes were assumed, the subsidence at the center of the cavity increases rapidly with increasing radii of the cavity. The subsidence calculated is 0.62, 4.77, and 16.9 feet for the cases with the radius of the cavity equal to 1000, 2000, and 3000 feet, respectively. For the case with the radius of the cavity equal to 2000 feet and assuming no weak bedding plane in the rock mass, the subsidence was computed to be 1.62 feet. The maximum subsidence observed in the field was 2.7 feet. It is very interesting to note by examining the observed and calculated subsidence profiles presented in Figure 7 that the observed subsidence could have been predicted if the radius of the cavity were assumed to be about 1500 feet which is about the size of the cavity expected in the field and three slip layers were assumed to exist. (The mined area is approximately 2000 feet by 4000 feet.) The calculated results also indicate that the subsidence profiles computed by assuming the existence of weak bedding planes are quite similar in shape to that observed in the field.

Case II

Geological conditions. The rock formation at the site is primarily shale with scattered thin layers of sandstone found at the depth of 110 to 1140 feet below the surface. The upper 1000-foot-thick layer is dark gray, green, and brown, laminated shales of the Bridger Formation. Below the Bridger Formation is 240-foot-thick, grayish-brown sandy Laney Shale, member of the Green River Formation. Immediately below the zone are predominantly light grayish green shale of the Wilkins Peak Member of the Green River Formation with a thickness of 480 feet. Be-

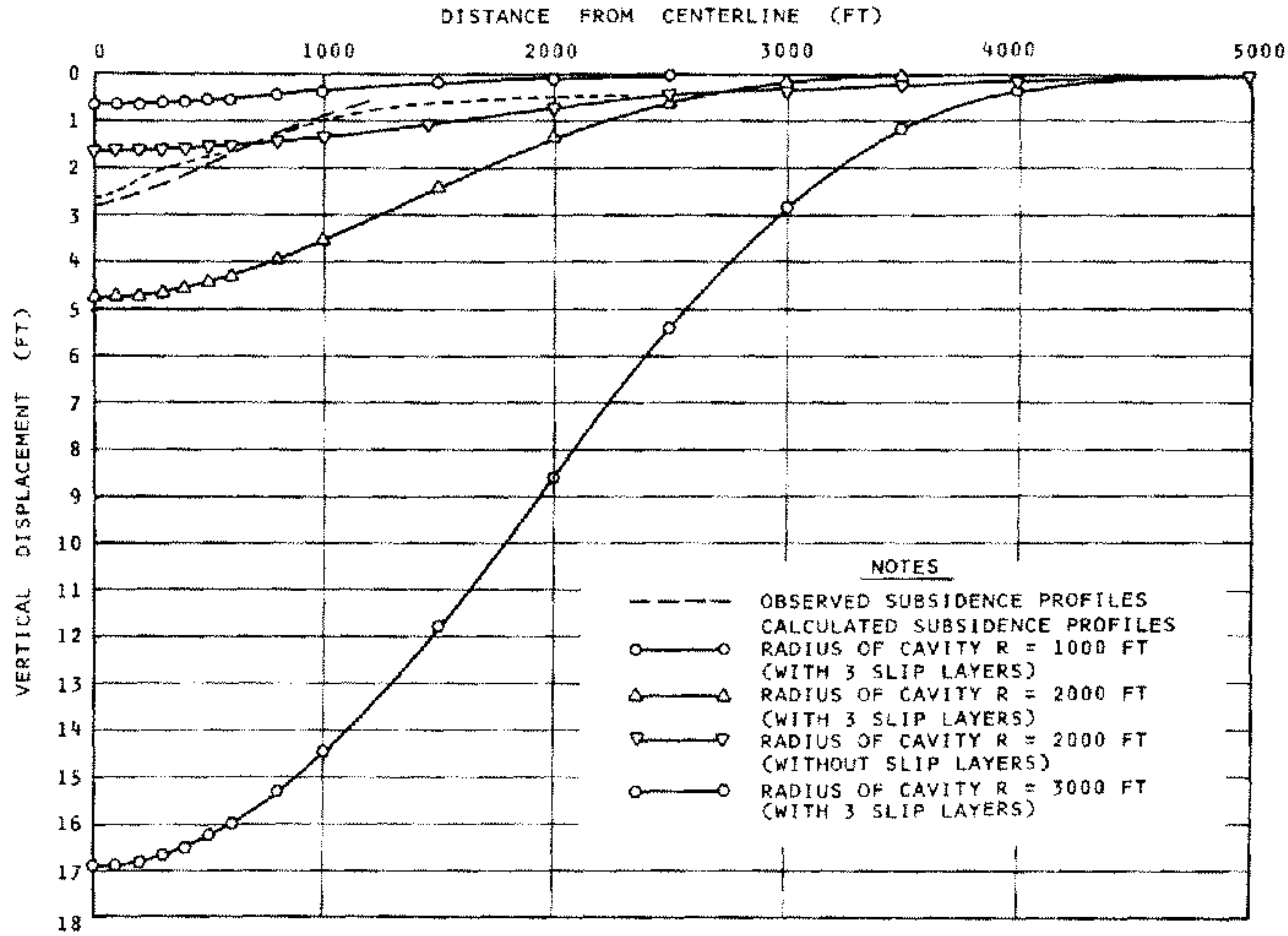

Figure 7. Subsidence profiles for various radii of a disc-shaped cavity, Case I.

TABLE II

Observed and computed surface subsidence—Case I

Radius of Cavity (ft.)	No. of Weak Bedding Planes	Subsidence (ft.)
1000	3	0.62
2000	None	1.62
2000	3	4.77
3000	3	16.9
Observed		2.7

low the Wilkins Peak Member is the Tipton Member of the Green River Formation composed of interbedded sandstones and brown shales. Scattered layers of trona were found to be located within the Wilkins Peak Member. The main seam mined is situated at the depth of 1557 to 1567 feet. The idealized rock profile with approximate values of the rock properties obtained from 3-D velocity logs is shown in Figures 8 and 9. In Figures 8, and 9, 7 and 11 possible weak bedding planes were assumed, respectively, within the rock mass. The locations of these weak planes were selected wherever there is a drastic change in the dynamic modulus obtained from 3-D velocity logs. It should be noted that because of the nature and type of rock present at the site, it is possible that a larger number of weak planes may be expected in the field.

Geometry of cavity. As shown in Figures 8 and 9, an average height of the cavity was 8 feet. An equivalent area of the cavity was assumed to be 90% of the total area mined. Because the shape of the actual cavity is a square, for the purpose of analysis, the cavity was assumed to be a circular disk with the same area as the equivalent cavity. The radius of the cavity was calculated to be 1080 feet. Four problems as summarized in Table III were analyzed. Problems 1, 2, and 4 were examined with the cavity 8 feet thick, and with none, 7 and 11 weak planes, respectively, present in the rock profile. Problem 3 was analyzed assuming the cavity is 15 feet thick and with seven weak planes in the rock mass to study effects of cavity thickness on the calculated subsidence.

Observed subsidence profile. As much as 5.5 feet of subsidence was measured near the centerline of the cavity.

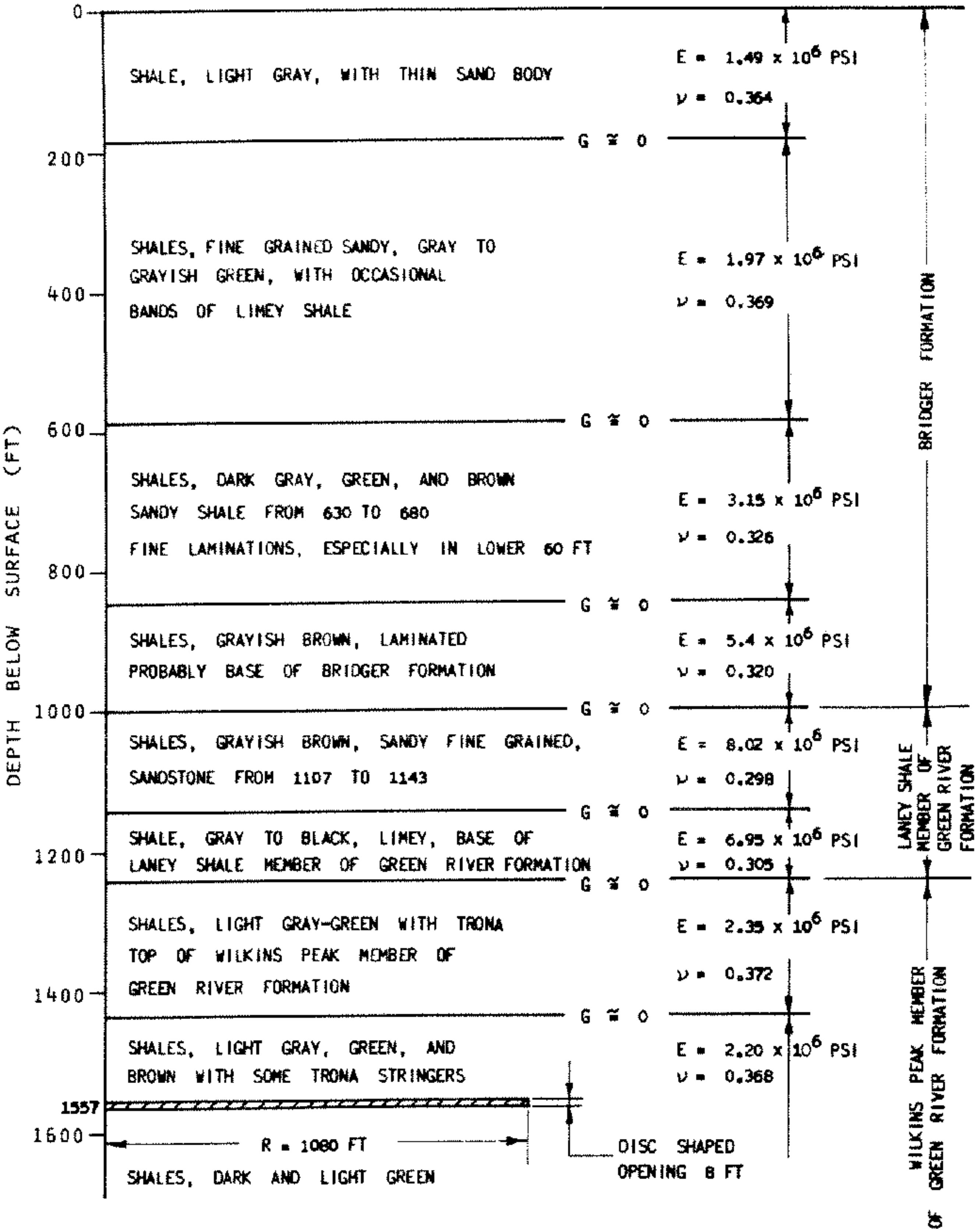

Figure 8. Idealized rock profile, Case II (7 bedding planes).

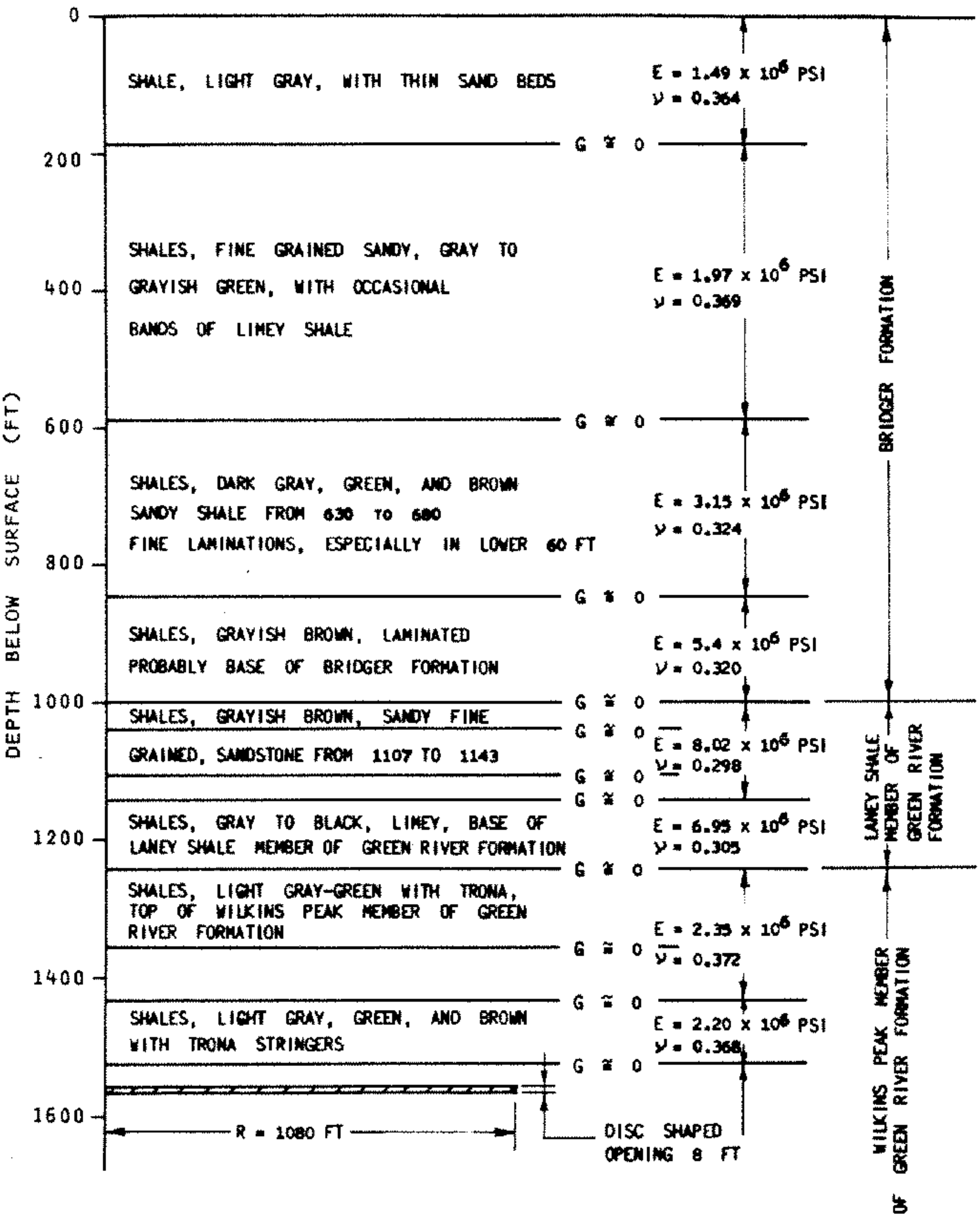

Figure 9. Idealized rock profile, Case II (11 bedding planes).

TABLE III

Problems analyzed—Case II

Problem	Height of Cavity (ft.)	Radius of Cavity (ft.)	Slip Layers	K_o	γ pcf
1	8	1080	None	1.0	136.2
2	8	1080	7 layers	1.0	136.2
3	15	1080	7 layers	1.0	136.2
4	8	1080	11 layers	1.0	136.2

The subsidence becomes smaller as the distance from the center of the cavity increases.

Comparison of computed and observed subsidence profiles. The calculated subsidence profiles for various cases analyzed, as well as the observed, are presented in Figure 10 and Table IV. For a cavity with a height of 8 feet, the maximum surface subsidence occurring at the center of the cavity was calculated to be 0.39, 2.98, and 3.40 feet for the cases with none, 7, and 11 assumed weak bedding planes, respectively. For the case with the cavity 15 feet in height, the subsidence is essentially the same as that of the cavity 8 feet high for the same number of the weak bedding planes assumed. This is to be expected because the cavity is situated at the great depth (1560 feet) below the surface, relative to the height. When the case with seven weak bedding planes was iterated once for no-tension analysis described previously, the subsidence within the mined area increased ten percent. It is expected that the subsidence will increase with increasing numbers of the iteration for no-tension analysis. As shown in Figure 10 when the number of weak bedding planes were increased from seven to eleven, the calculated subsidence increased from 2.98 to 3.40 feet. Because of the type of rock (primarily shale) present at the site, larger numbers of weak bedding planes may be present in the strata. Thus, the subsidence may be expected to be larger within the mined area.

The observed subsidence was 5.5 feet at the time of survey in November, 1969. The modulus values used in the computation were based on 3-D velocity logs. Considering the overall rock mass, it is generally agreed that these values are probably on the high side by a factor which may be as high as two. With a modulus of one-half

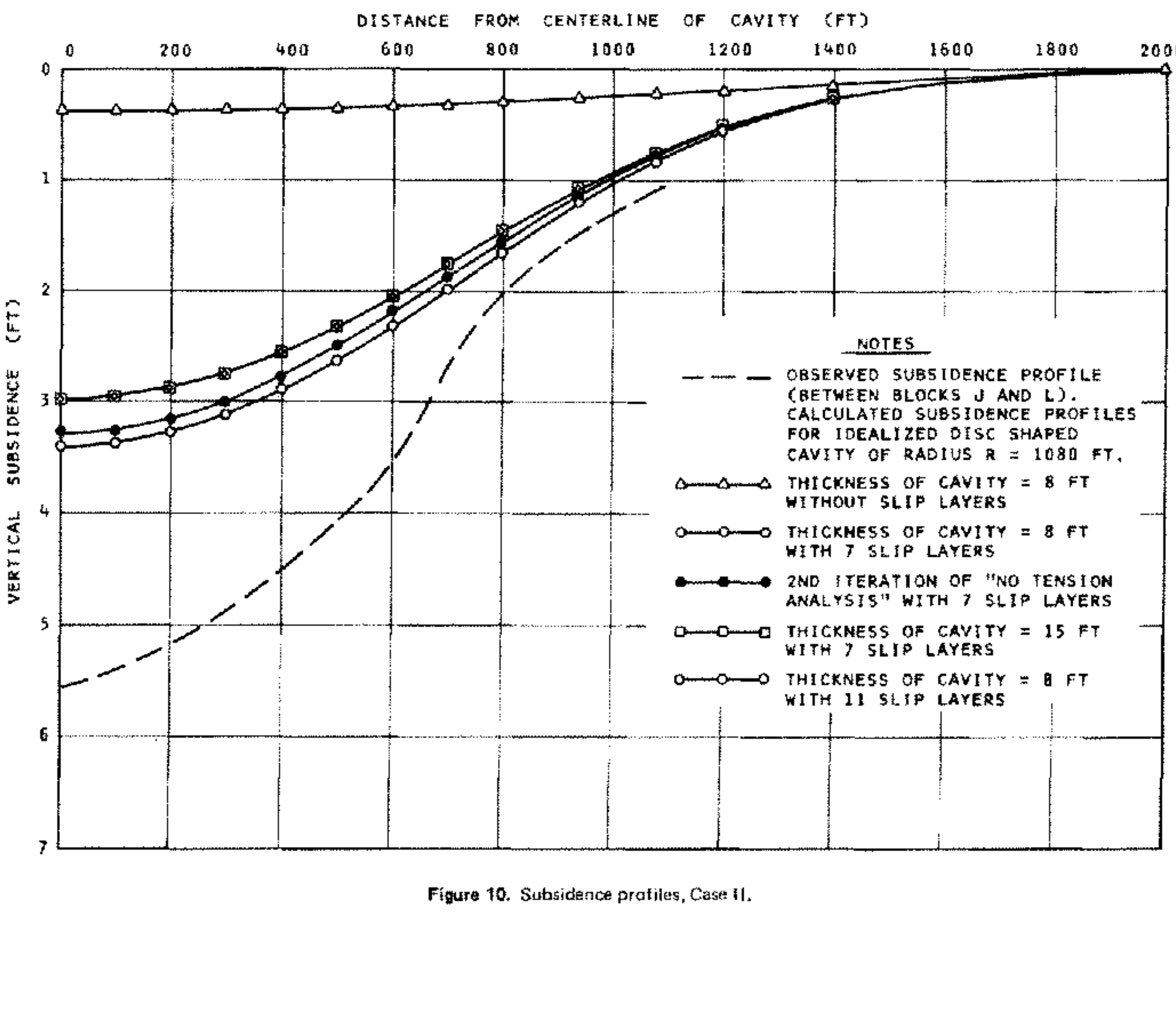

Figure 10. Subsidence profiles, Case II.

TABLE IV

Computed and observed surface subsidence—Case II

Radius of Cavity (ft.)	Height of Cavity (ft.)	No. of Weak Bedding Planes	Subsidence (ft.)
1080	8	None	0.39
1080	8	7	2.98
1080	8	7	3.29*
1080	8	11	3.40
1080	15	7	2.99
Observed			5.50

*After one iteration for no tension analysis

that used in the analysis, the computed subsidence would be in close agreement with the measured values. The discrepancy between the calculated and observed subsidence may also be improved if a larger number of the weak bedding planes is assumed or a larger number of iterations is performed for the no-tension analysis.

The calculated horizontal strain is illustrated in Figure 11. The horizontal strain distribution is quite similar to that observed. A larger horizontal compression strain was calculated at the center of the mined area. The horizontal compressive strain decreases rapidly and changes to tensile strain as the distance away from the center of the mined area increases. The tensile strain was calculated to be maximum at a short distance outside the mined area and then decays rapidly with the distance away from the mined area.

Because of the presence of the weak bedding planes, relative horizontal movements as much as 0.85 feet were calculated across these bedding planes. Figure 12 shows the horizontal displacement calculated at two vertical sections 500 and 1200 feet from the center of the mined area. Several features should be noted: (1) each drastic change in horizontal displacement is associated with the location of a bedding plane, (2) larger relative horizontal displacements occur near the surface, and (3) the relative horizontal displacements occurring within the bedding planes decrease with increasing distance from the center of the mined area.

Figure 13 illustrates calculated vertical displacements along the centerline of the mined area for the cases with none and 11 weak bedding planes. It may be noted that the vertical displacement increased slightly with increasing depth below the surface. Movements above mined areas similar to those calculated have been reported by Mohr (1958). An example of this is shown in Figure 14.

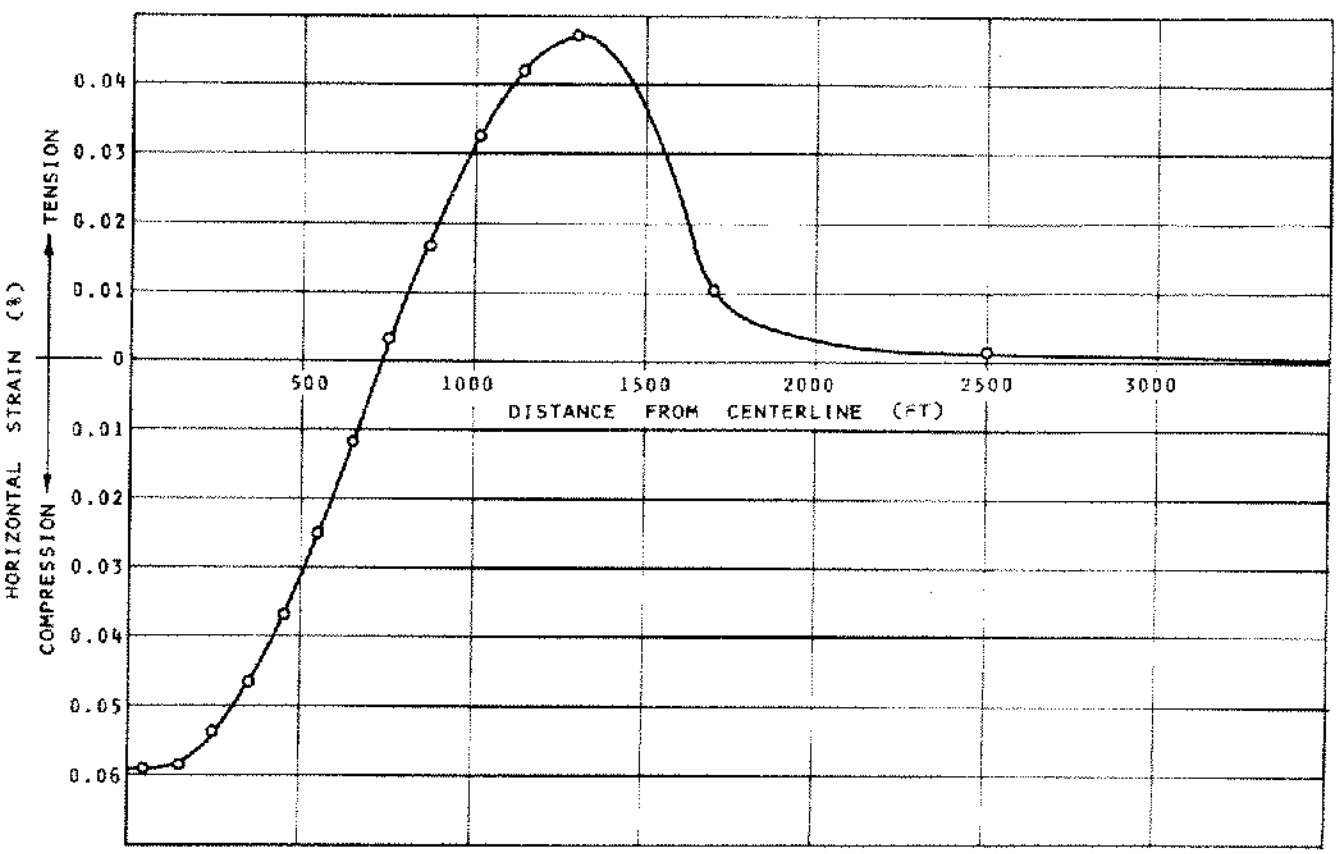

Figure 11. Percent horizontal strain calculated for Case II (t = 8 ft., 11 weak bedding planes).

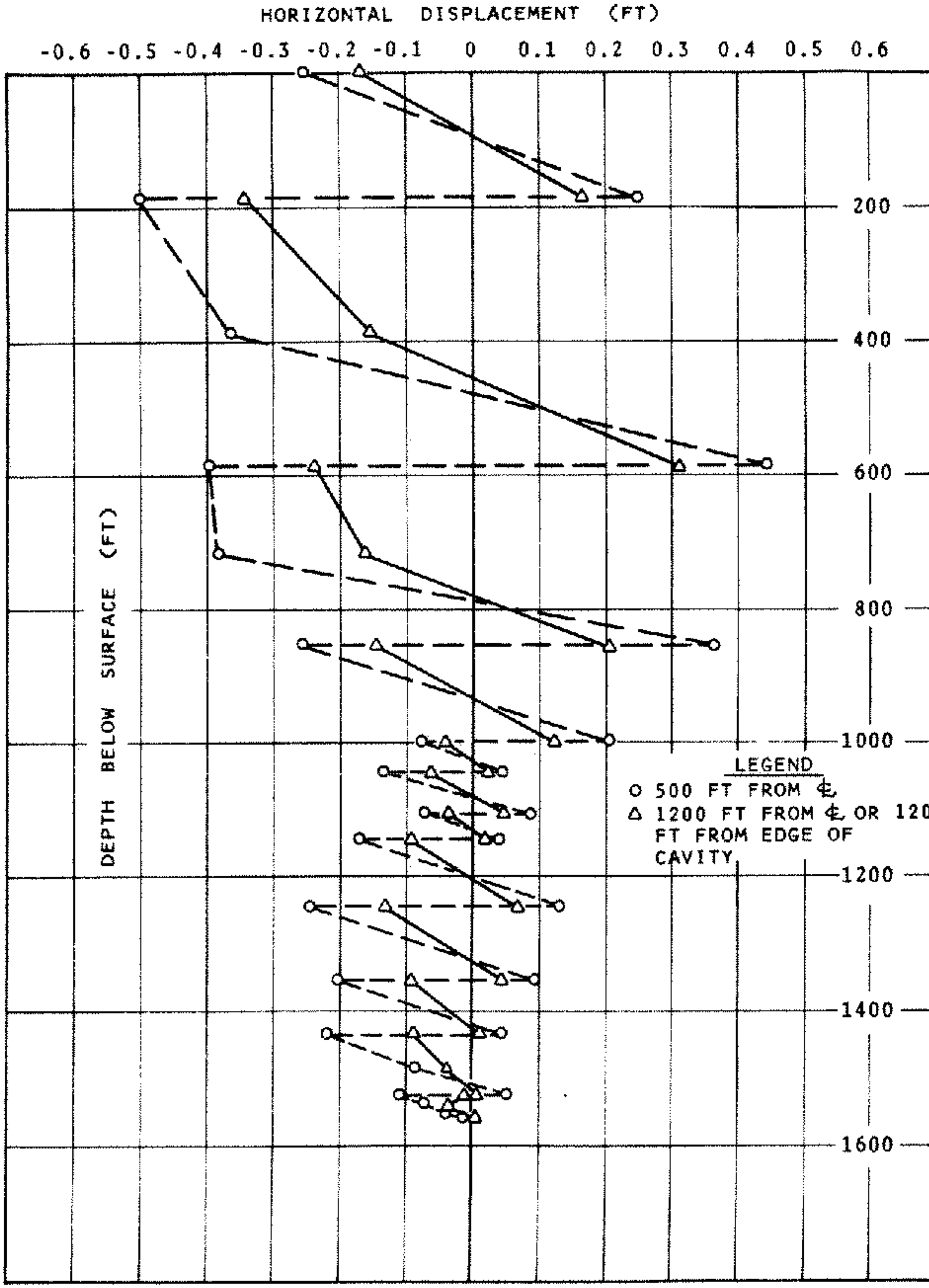

Figure 12. Horizontal displacement calculated at various vertical sections, Case II (t = 8 ft., 11 weak bedding planes).

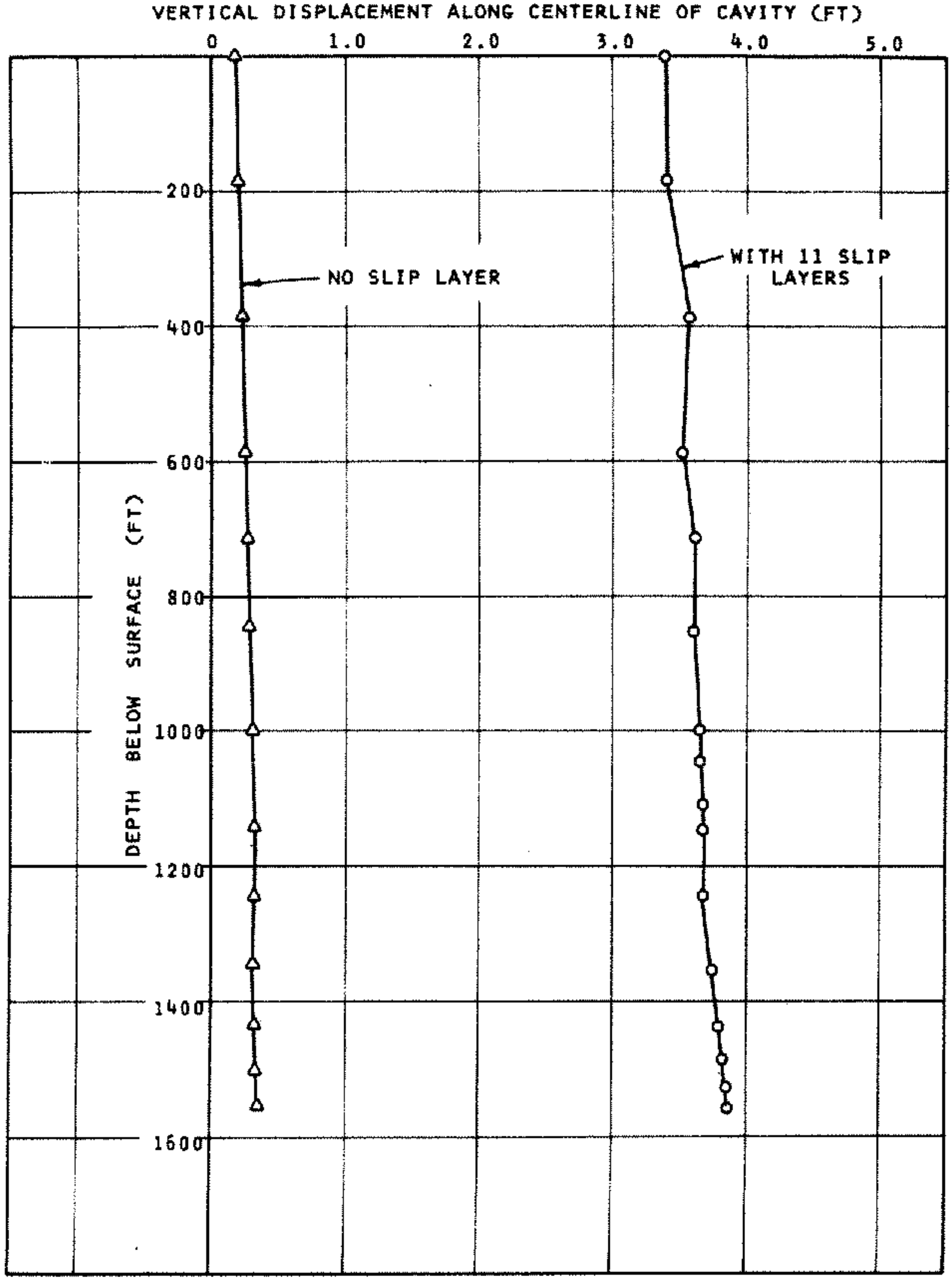

Figure 13. Vertical displacement calculated along the centerline of the mined area, Case II (t = 8 ft., 11 weak bedding planes).

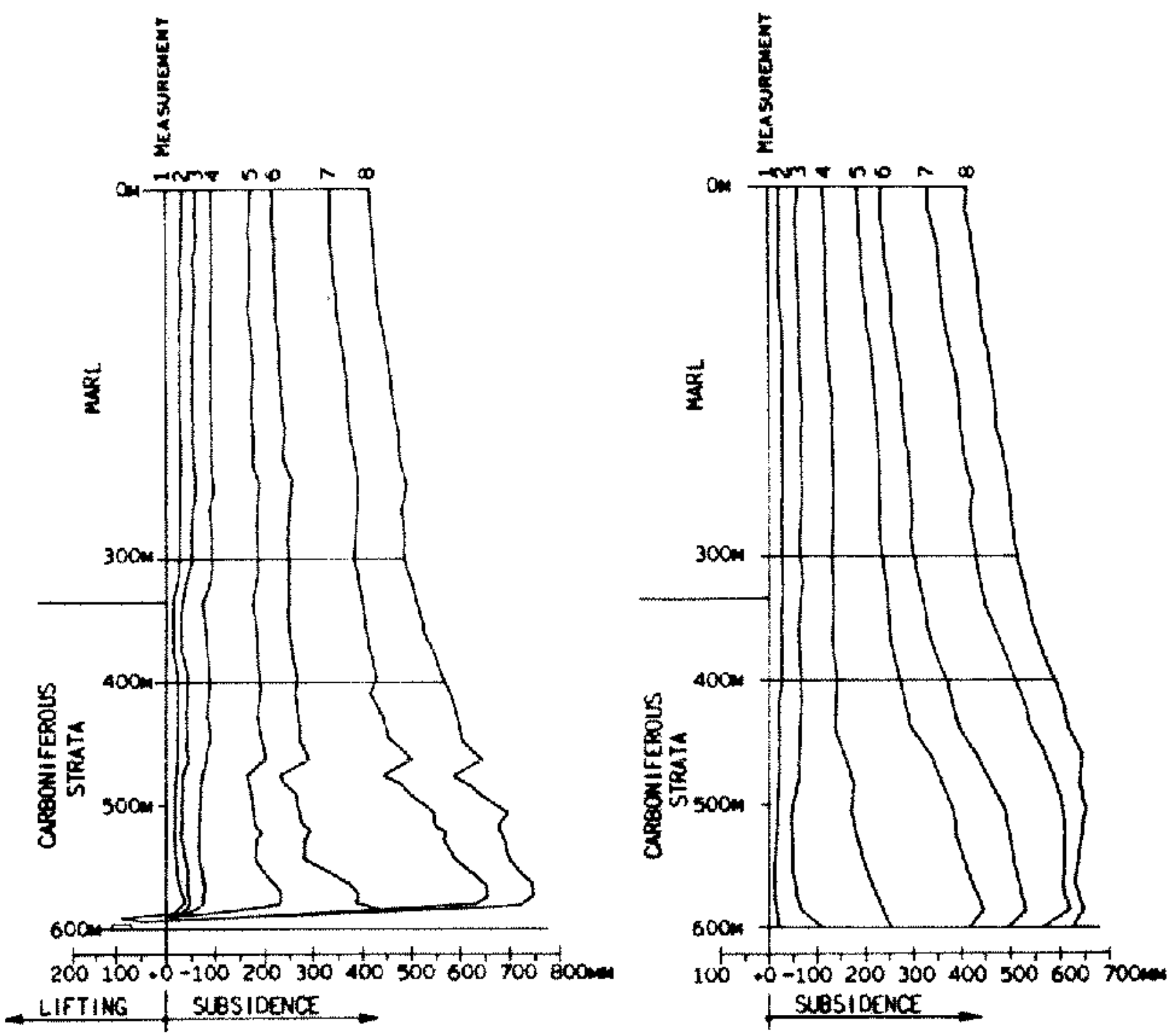

Figure 14. Subsiding of the rock above a working (after Mohr, 1951).

SUMMARY AND CONCLUSIONS

The influence of failure in the rock mass surrounding a cavity on the surface subsidence was investigated in this study. Two possible modes of failure were considered: (i) compressive yielding, and (ii) failure under tensile stresses. Based on results of these investigations and other studies, it was concluded that the occurrence of this type of failure in the proximity of the opening does not have a significant influence on the surface subsidence. However, this type of failure can propagate; and should the failure progress to the surface, then the massive subsidence associated with sinkholes and ground breakage can occur. Therefore, the development of a computational technique which can investigate the possibility of rock failure progressing to the surface is an important aspect in designing mining operations to limit subsidence.

For the majority of practical problems, failure does not propagate to the surface, and it is necessary to predict the subsidence that might occur from the creation of solution cavities. A computational technique which permits slippage along bedding planes and other geological continuities has been developed. Based on a comparison with observed subsidence in two case histories, it was found that if slippage was permitted along bedding planes, there was reasonable agreement between measured and computed values.

Based on the results of this and previous investigations, it can be concluded that the analytical methods presented in this report have the potential of predicting the subsidence above mined areas provided there is adequate information on the geologic conditions.

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REFERENCES

Deere, D. U. (1961). Subsidence due to mining—a case history from the Gulf Coast Region of Texas. *Proc. Fourth Symposium on Rock Mechanics*, Penn. State Univ., Pa., pp 59–64.

______Hendron, A. J., Jr., Patton, F. D. and Cording, E. J. (1966). Design of surface and near-surface construction in rock. *In* Failure and Breakage of Rock *Proc. Eighth Symposium Rock Mechanics*, 1966, Amer. Inst. Mining Eng., New York, pp 237–302.

Drucker, D. C. (1952). A more fundamental approach in plastic stress-strain relations, *Proc. First U.S. National Congress Applied Mechanics*, New York, pp 487–491.

Lee, K. L. and Strauss, M. E. (1969). Prediction of horizontal movements due to subsidence over mined areas. *International Symposium on Land Subsidence*. Intern. Assoc. Hydrolog. Sci., Tokyo, Japan, 2:512–528.

Mohr, F. (1951). The effect of extraction on the roof with special reference to the action on shafts. Bergb.-Arch, 14:(1), 1.

______(1958). Observations in shafts on rock movements due to mining, *International Strata Control Congress*, Leipzig, pp 112–123.

Nair, K., Singh, R. D., Chang, C-Y, and Abdullah, A., 1974. Time-dependent analysis to predict closure in salt cavities. This symposium.

Nair, K. and Chang, C-Y (1969a). Investigation of the influence of certain variables on the subsidence above mined areas. Report to Solution Mining Research Institute, Inc., unpublished.

______(1969b). Stability studies for Project Payette. Final Report submitted to Fenix and Scisson, Tulsa, Oklahoma, unpublished.

Patton, F. D. (1966). Multiple modes of shear failure in rock and related materials, PhD thesis, University of Illinois, 1965, p. 232, Abstracted in *Proc. First Congress International Society of Rock Mechanics*, Lisbon (Lisbon: Laboratorio Nacional de Enginharia Civil, 1966), 1:509–513.

Reyes, S. F. and Deere, D. U. (1966). "Elastic-plastic analysis of underground openings by the finite element method," *Proceedings, First Congress of the International Society of Rock Mechanics*, Lisbon, Portugal.

Zienkiewicz, O. C., Valliappan, S., and King, I. P. (1968). Stress analysis of rock as a "no-tension" material. *Geotechnique* 18, (1), 56–66.